

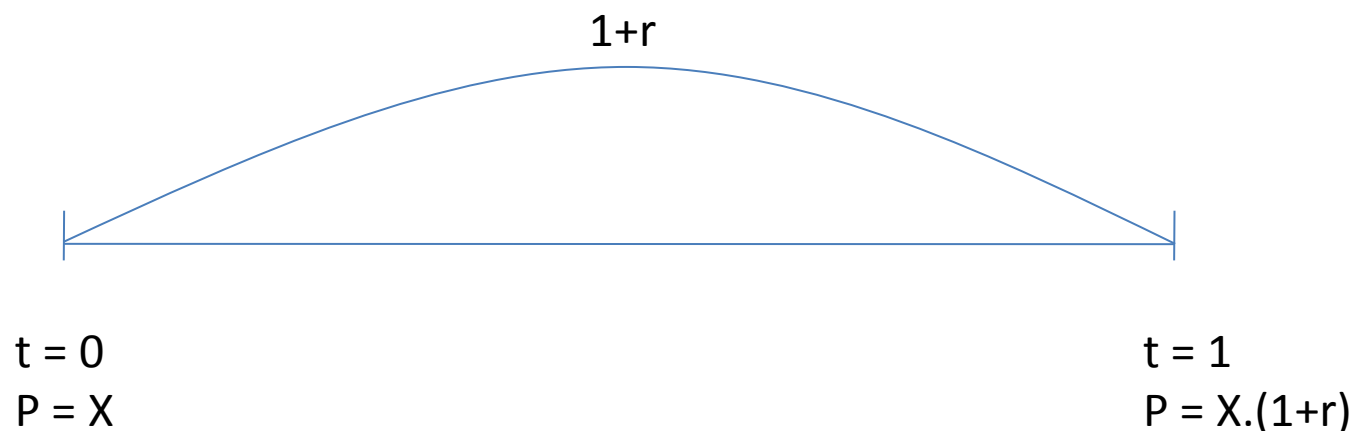
I. Time Value of Money; discounting

“Jam today is better than jam tomorrow!”

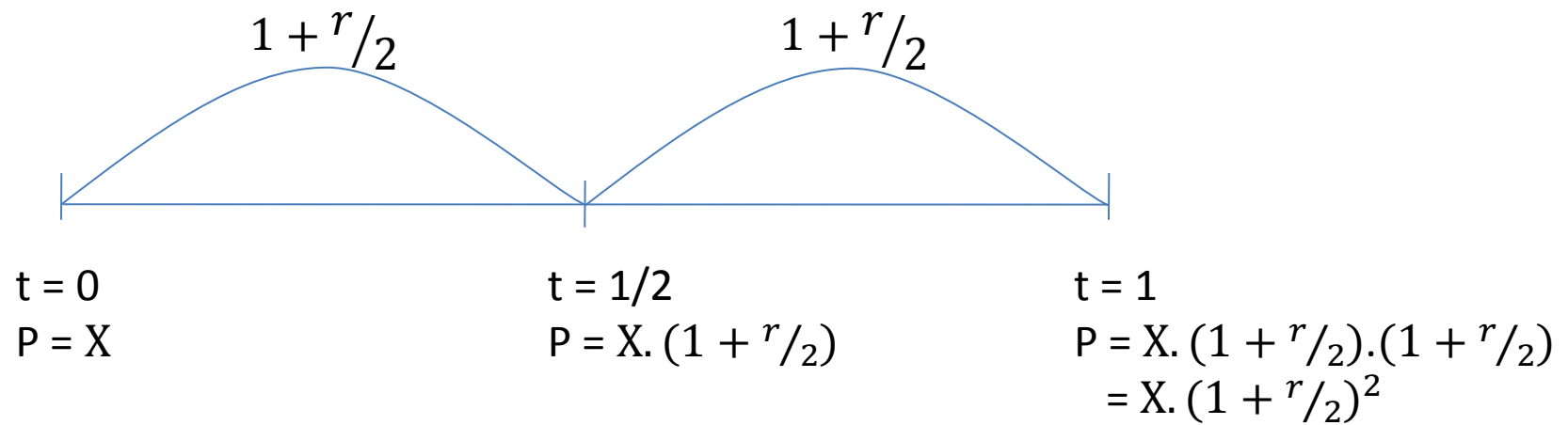
1. Simple interest rate:

Let r be the simple *yearly interest rate*, i.e. with X units of pound invested for one year, at the end of the year we have:

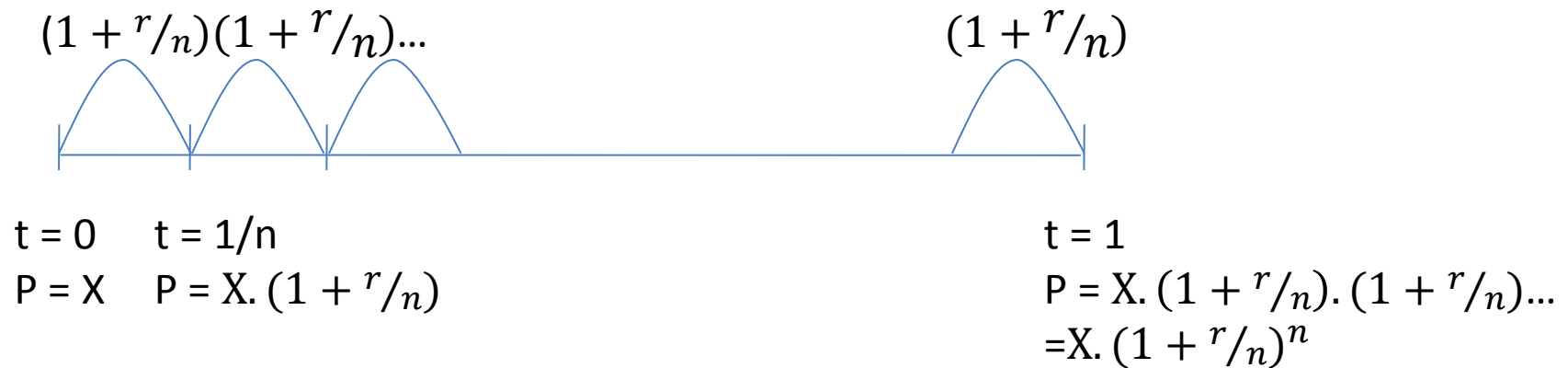
- with interest calculated yearly:



- with interest calculated twice yearly:



- with interest calculated (compounded) n times per year:



- Compound interest (interest on the interest)

We use:

$$\left(1 + \frac{x}{n}\right)^n \uparrow e^x \quad (n \rightarrow \infty) \quad (x > 0)$$

“Exponential growth is the limit of compound interest as the interest is compounded continuously”.

Notes on Exponential growth:

1. Occurs widely in nature and in finance, e.g. Growth of biological populations, radioactivity decay, computational complexity theory, etc...
2. It is extremely rapid.
3. Without interest it is difficult to justify taking the risk of lending money.

2. Discounting

- Money is lent and borrowed at interest - hence one must distinguish between: price in nominal terms and prices in real terms.
- We do this by discounting: one discounts over a time interval of length t by multiplying prices by the factor

$$e^{-rt}$$

where the quantity r is called the *short rate* or *continuously compounding rate* as it is the interest rate for investing over very short periods of time.

- **Examples:**

If S_T denotes a price at later time T , its real value at time t is:

$$S_T \cdot \exp\{-r \cdot (T - t)\}$$

- **Question 1:** If today I put \$200 in an account with short rate $r = 1\%$, how much money will I have in 3 years?
- **Question 2:** How much money should I put in an account that pays me 3% yearly interest in order to have: \$100 in 1year? \$200 in 4 years?

- In all our calculations below we will discount everything.
- Discounting is done for accounting purposes in **Net Present Value** (NPV) calculations:

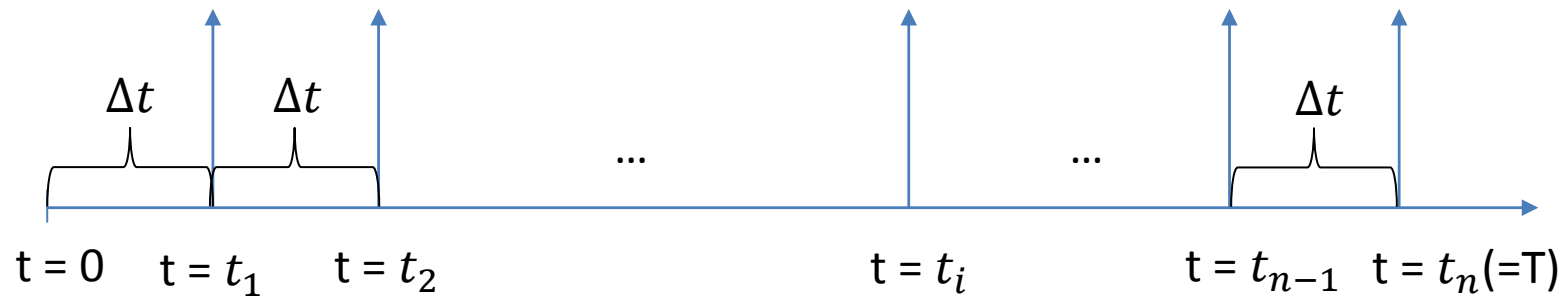
Each cash inflow/outflow is discounted back to its present value (PV), then they are summed.

So: NPV compares the present value of money today to the present value of money in future, taking inflation and returns into account.

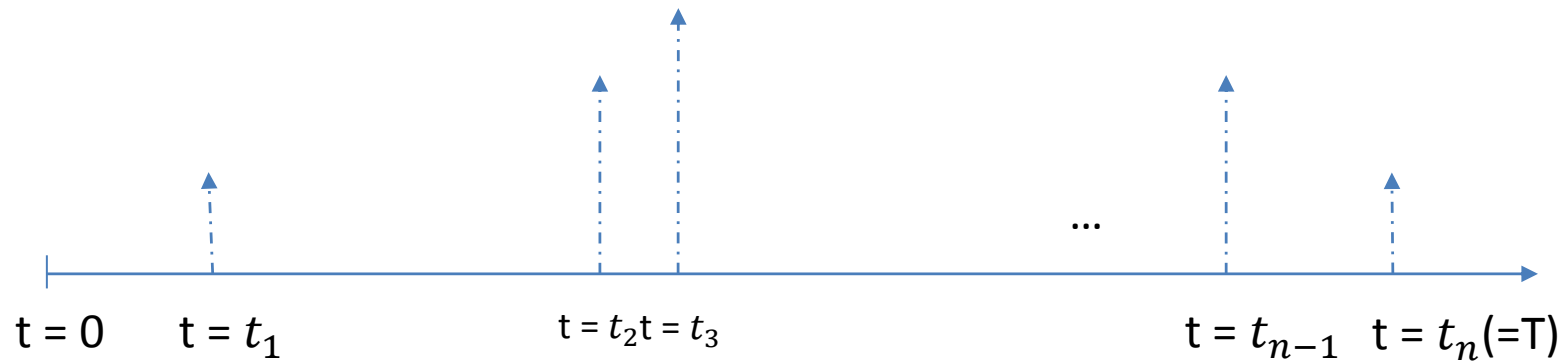
- These are also used as a tool in assessing whether or not an investment should be made.

- Illustrating NPV calculation

1) Fixed payments at regular time interval:



2) Non-periodic, variable payments:



Notes on interest rates:

1. Borrowing vs. lending – Bank lend and borrow from each other: London Inter-Bank Offer Rate (**LIBOR**).

It is the average interest rate estimated by leading banks in London that they would be charged if borrowing from other banks.
Primary benchmark for short term interest rates around the world.

2. Risky vs. riskless - Major Government bond (promising to pay a certain amount - the face value - on a certain date, as well as periodic interest payments) are traditionally regarded as riskless.

Bonds are debt investments whereby an investor loans a certain amount of money, for a certain amount of time, with a certain interest rate, to a company or country.

3. Fixed or variable rate, $r(t, T)$.
4. Stochastic or deterministic.
5. Term structure of interest rates.